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Abstract

The empirical literature in corporate finance has documented robust findings regarding leverage, dividend, and investment decisions that appear inconsistent with the predictions of leading theories. For instance, many papers report negative relations between profitability and leverage, and between dividends and investment-cash flow sensitivity. We derive a dynamic model of the firm that is able to rationalize the main empirical findings in a unified way. In addition, we successfully explain the existence of all-equity firms and its observed characteristics. We justify the empirical regularities by acknowledging the endogeneity of the variables under study as well as their dependence on the primitive characteristics of the firm.

JEL classification: G31, G32

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1 Introduction

During the last decades, the empirical literature in corporate finance has reported a few regularities regarding leverage, dividend, and investment decisions. Many of these results have been used to question the validity of central theories whose implications were found to be inconsistent with the empirical evidence. For instance, the negative relation between profitability and leverage, which has been documented in numerous studies, has been used to cast doubt on the trade-off theory of capital structure, and to support alternative, competing theories, such as the pecking order model of financing decisions.[1] As another example, the fact that firms that pay lower dividends exhibit stronger investment-cash flow sensitivities has been used to criticize the assumption of perfect capital markets in the context of the neoclassical intertemporal model of investment.[2] We propose a simple dynamic model of the firm that is able to rationalize the main empirical findings, providing plausible justifications for all of them.

In the dynamic model we propose, investment and leverage decisions are the choice variables of the firm, while dividends arise endogenously as a residual of the other two. More precisely, the firm decides how much to invest for the next period, as well as how to finance those assets (i.e., with debt and/or equity), depending on current profit shocks and its own primitive characteristics. We find that two firm characteristics are of particular importance in explaining firm behavior, namely, the curvature of the production function and non-debt tax shields (e.g., depreciation, operating costs, and R&D expenses).[3] The influence of these features is such that they often overwhelm the effect of the others and explain most of our results. In order to replicate the main empirical findings, we create a panel of firms that differ regarding primitive characteristics and simulate their choices by using our model. We then construct the variables of interest (e.g., dividends, profits, etc) from those decisions. Finally, we pool the constructed variables for the different types of firms in the panel and run the standard regressions used by the empirical literature in corporate finance. Our results are in line with the empirical regularities and the

[1] See, e.g., Fama and French (2002) for a recent discussion on this and other related issues.

[2] See, e.g., Fazzari, Hubbard, and Petersen (1988).

[3] The industrial organization literature in economics has extensively studied the curvature of the production function (see, e.g., Ackerberg, Benkard, Berry, and Pakes (2007), Ackerberg, Caves, and Frazer (2006), and the references therein).

model we use allows us to offer structural justifications for each of them. We next describe our main findings.

We start studying leverage and dividend decisions. The standard, static version of the trade-off model predicts that more profitable firms should have more leverage. The empirical evidence shows almost unanimously that the opposite result holds in practice (e.g., Long and Malitz (1985), Titman and Wessels (1988), Rajan and Zingales (1995), Fama and French (2002)). The dynamic model we offer can accommodate this result, which is the seemingly most important inconsistency of the trade-off theory of capital structure. Specifically, we run the corresponding regression with data simulated from our model and find a significantly negative coefficient on profitability in leverage decisions. Briefly, the reason is as follows: When leverage and investment decisions are simultaneously made, profitability arises endogenously as a consequence of those choices. Moreover, we find that those firm characteristics that have the largest impact on book and market leverage (i.e., the curvature of the production function and non-debt tax shields) are the same features with the greatest effect on profitability. In addition, the effect of those characteristics on book and market leverage is opposite in sign to that on profitability. Then, firms with combinations of features that make them highly profitable tend to have, at the same time, low book and market leverage and vice versa. When these firms are pooled in a single regression, the negative coefficient on profitability naturally occurs.

The dynamic model of the firm we offer can also rationalize several other observed results about leverage and dividends in the cross-section of firms (Fama and French (2002) contains a comprehensive summary of these findings). For instance, the empirical work has noted that corporations with higher dividend payouts are more profitable and invest less. Furthermore, it has reported that firms with more leverage have lower dividend payouts, and that firms with more investment opportunities (i.e., higher market-to-book ratio) and higher volatility of profits have lower leverage and dividends. Finally, the empirical evidence suggests that leverage is negatively associated with non-debt tax shields.[4] Results in our model are consistent with all these observations and the structural explanations follow the same argument as above. That is, the

[4] The model developed by DeAngelo and Masulis (1980) predicts a negative association between leverage and non-debt tax shields.

elasticity of capital and non-debt tax shields are the dominant firm characteristics, driving these endogenous variables consistently with the observed evidence. More importantly, these results emerge in a simple dynamic model with no agency costs or asymmetric information problems, which are the usual assumptions invoked to justify most of these relationships. The former include Fama and Miller (1972), Jensen and Meckling (1976), Myers (1977), Easterbrook (1984), and Jensen (1986), while the latter include Myers and Majluf (1984).

We close the study of leverage and dividend decisions by addressing one of the most puzzling empirical findings, that is, the existence of all-equity firms. This phenomenon is extensively documented by Strebulaev and Yang (2013), who report that an average of around 10% of large public nonfinancial U.S. firms have had zero debt in the last decades. They also find that those firms pay relatively higher taxes and dividends, are more profitable, and have higher market-to-book ratios. Furthermore, they suggest this observation is independent of firm size. Our dynamic model can replicate all these findings, shedding light on this long-standing puzzle. The structural explanation is that certain combinations of primitive characteristics give rise to firms making this type of decisions. More specifically, firms with high non-debt tax shields will have low, possibly zero, leverage. When those firms also have low elasticity of capital, they turn out to be highly profitable, have high market-to-book ratios, and pay large dividends and taxes. Finally, because these results arise under the normalization of the parameter that regulates firm size (the drift of the profit shock process), the previous results hold irrespective of how large the firm is.

We finally study investment decisions. One of the most prominent results is the empirical observation that firms perceived a priori as more financially constrained, exhibit stronger investment-cash flow sensitivities, even after controlling for marginal q (e.g., see Fazzari, Hubbard, and Petersen (1988), Hubbard (1998), and the references therein). This result has been used to criticize the neoclassical model of investment as it suggests that capital market imperfections are necessary to explain observed investment behavior. Our dynamic model is able to rationalize this result even in the context of a perfect capital market or, in other terms, without financial constraints. We do the usual regression of investment ratio on internal cash flow and market-to-book ratio employing the simulated cross-section of firms and find that, indeed, the coefficients on internal cash flow and market-to-book ratio are significantly positive. Most importantly, we find that firms that pay a smaller proportion of net income as dividends (i.e., the candidates to be considered more financially constrained) have larger investment-cash flow sensitivities, and vice versa. The force behind this result is that firms with high elasticity of capital, the dominant characteristic for these endogenous variables, turn out to have low dividend ratios and high investment-cash flow sensitivies. Furthermore, most of the other parameters affect these variables in opposite direction, which reinforces the negative association between them.

Last, but not least, we relate our work with existing dynamic programming models of the firm. Hennessy and Whited (2005) rationalize the negative relation between leverage and profitability with a dynamic model that features financial transaction costs. Using a time-series approach, they show that leverage is negatively related with profitability and lagged cash flow, as well as with an external finance weighted average q. Similarly, Tserlukevich (2008) uses a dynamic real options model emphasizing on irreversibility and fixed costs of investments. He finds that leverage is negatively associated with profitability, mean-reverting, and path dependent. In addition, Moyen (2004) uses a dynamic model to reconcile the contradictory empirical evidence presented by Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997).[5] In order to achieve that goal, she compares the investment-cash flow sensitivity for two different types of firms: those that suffer financing constraints and have no access to external capital versus unconstrained firms that can access external funds. We contribute to this literature by rationalizing the empirical regularities explained by these papers with a dynamic programming model that features no frictions neither on the real nor the financing side. Instead, we search for the answers by emphasizing on the link between the endogenous variables under study and the primitive characteristics of the firm in the cross-sectional analysis.

The paper is organized as follows. In Section 2, we derive a dynamic model of the firm in closed-form. In Section 3, we describe the model predictions regarding leverage, dividends, and investments, and explain the structural forces underlying them. Section 4 concludes. Appendix 1 describes briefly the proof of the solution of the problem of the firm, while Appendix 2 contains the sensitivity analysis of the main model variables.

[5] Kaplan and Zingales (1997) report findings suggesting that investment-cash flow sensitivities cannot be used to identify firms subject to financing constraints.

2 A Dynamic Model of the Firm

This section describes the dynamic model of the firm as well as its explicit solution.[6] We then explain how the optimal decisions of the firm depend on its primitive characteristics.

We use discrete-time, infinite-horizon, stochastic dynamic programming to solve the problem of the firm. Thus, the life horizon of the firm is infinite and the CEO makes decisions at the end of every period (e.g., quarter, year, etc.) in order to maximize the stock price. Our model includes two fundamental features that enhance the existing dynamic programming models of the firm in corporate finance. First, we introduce long-run growth, which could be interpreted as the possibility of the firm to take advantage of new profitable investments in the future. (We write a tilde on variable $\widetilde{X}$ to indicate that the variable is growing over time.) Second, the model is based on the separation principle, which states managers maximize shareholders' wealth by undertaking the investments that maximize firm value, independently of equityholders' personal preferences. It follows that our model does not require any assumption about shareholders' utility functions, as long as we discount future cash flows with an appropriately risk-adjusted discount rate.[7]

The firm has two variables of choice: capital and debt. The book value of assets in period t is indicated by variable $\widetilde{K}_t$. The capital of the firm $\widetilde{K}_t$ is used for production and varies (i.e., increases or decreases) over time because of investment decisions. Firm assets depreciate at constant rate $\delta > 0$ in each period. Variable $\widetilde{D}_t$ represents the book value of debt in period t. We assume debt matures in one period and is rolled over at the end of every period. Furthermore, as a means to simplify the analysis, we assume the coupon rate c_B equals the market cost of debt r_B, which implies that book value of debt $\widetilde{D}_t$ equals market value of debt $\widetilde{B}_t$. The firm increases and decreases the amount of outstanding debt $\widetilde{B}_t$ over time as needed in order to maximize the market value of equity. In the spirit of DeAngelo, DeAngelo, and Whited (2011), we assume the firm keeps debt risk-free over its life, so that it can always repay its debt in full. This limited debt capacity assumption can be motivated from the literature on credit rationing in the context of imperfect information (see, e.g., Jaffee and Russell (1976) and Stiglitz and Weiss (1981)), and

[6] The model we study is similar to the one used by Lazzati and Menichini (2013).

[7] See, for example, Copeland, Weston, and Shastri (2005) for a more complete discussion of the separation principle.

helps us to rationalize the observation of debt conservatism reported by Graham (2000). Then, in our model, the market cost of debt r_B equals the risk-free interest rate r_f.

There is one exogenous state variable that makes the model stochastic, the profit shock z_t. We assume profit shocks follow an AR(1) process in logs

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \varepsilon_t \tag{1}$$

where the autoregressive parameter $\rho \in (0,1)$ defines the persistence of profit innovations. When ρ is high, the periods of high profits (e.g., economic booms) and low profits (e.g., recessions) are longer on average, and vice versa. The innovation term ε_t is assumed to be an *iid* normal random variable with mean 0 and variance σ^2. The drift in logs $c > 0$ scales the moments of the distribution of z_t and plays an important role in the expected profits of the firm.[8]

Earnings before interest and taxes in period t are

$$\widetilde{E}_t = (1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t \tag{2}$$

where z_t is the realization of the profit innovation in period t and parameter $\alpha \in (0,1)$ represents the curvature of the production function. The factor $(1+g)^{t(1-\alpha)}$ can be interpreted as the level of technology available to the firm in period t and allows for a required normalization of growing variables that we describe in Appendix 1. This feature implies the firm can grow at constant rate $g \geq 0$ in each period. Equation (2) shows that the gross profits function (the first term) is of the Cobb-Douglas form with decreasing returns to scale in capital input. We assume that the secular growth rate is lower than the market cost of equity (i.e., $g < r_S$), a requirement needed to guarantee existence of market value of equity. The market cost of equity is exogenous and depends on the risk of the stock price. The last two terms of equation (2) are the non-debt tax shields and include the operating costs $f\widetilde{K}_t$ (with $f > 0$) and capital depreciation $\delta\widetilde{K}_t$ of the period.

Finally, taking into account that the firm pays income tax on corporate earnings at rate $\tau \in (0,1)$, net profits of the firm in period t are

$$\widetilde{N}_t = \left(\widetilde{E}_t - r_B\widetilde{B}_t\right)(1-\tau). \tag{3}$$

[8] It has been common in the literature to normalize parameter $c = 1$.

We can then define the accounting cash flow equation as

$$\widetilde{L}_t = \widetilde{N}_t - \left[\left(\widetilde{K}_{t+1} - \widetilde{K}_t \right) - \left(\widetilde{B}_{t+1} - \widetilde{B}_t \right) \right] \tag{4}$$

which represents the dividend paid by the firm to shareholders in period t. This dividend equals net profits minus the change in equity. Given the current state of the firm at $t = 0$, $\left(\widetilde{K}_0, \widetilde{B}_0, z_0 \right)$, the CEO chooses an infinite sequence of functions $\left\{ \widetilde{K}_{t+1}, \widetilde{B}_{t+1} \right\}_{t=0}^{\infty}$ that maximize the market value of equity. Accordingly, the stock price is given by

$$\widetilde{S}_0 \left(\widetilde{K}_0, \widetilde{B}_0, z_0 \right) = \max_{\left\{ \widetilde{K}_{t+1}, \widetilde{B}_{t+1} \right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{(1 + r_S)^t} \widetilde{L}_t \tag{5}$$

subject to the restriction of risk-free debt, and where E_0 denotes the expectation operator given information at $t = 0$ (i.e., $\widetilde{K}_0, \widetilde{B}_0, z_0$). We describe next the general solution of the dynamic model in closed-form. (Appendix 1 contains a sketch of proof for the optimal policies.) A direct benefit of obtaining closed-form solutions is that we can easily see how firm decisions depend on primitive characteristics.

Optimal Firm Decisions

$$\widetilde{K}^*_{t+1} (z_t) = (1 + g)^{t+1} E \left[z_{t+1} | z_t \right]^{\frac{1}{1-\alpha}} W^* \quad \text{and} \quad \widetilde{B}^*_{t+1} (z_t) = \ell^* \widetilde{K}^*_{t+1} (z_t) . \tag{6}$$

This expression shows optimal capital, $\widetilde{K}^*_{t+1} (z_t)$, and debt, $\widetilde{B}^*_{t+1} (z_t)$, for next period as explicit functions of firm characteristics. Factor $(1 + g)^{t+1}$ in $\widetilde{K}^*_{t+1} (z_t)$ denotes the accumulated growth since firm inception, while $E \left[z_{t+1} | z_t \right] = c z_t^{\rho} e^{\frac{1}{2} \sigma^2}$ is the conditional expectation of the profit shock in the next period given current shock. This factor introduces correlation across time in firm decisions. Variable W^* is given by

$$W^* = \left(\frac{\alpha}{(1 - \ell^*) \frac{r_S}{1 - \tau} + \ell^* r_B + f + \delta} \right)^{\frac{1}{1-\alpha}} \tag{7}$$

and denotes the part of optimal capital that does not depend on the current shock. In our model, optimal debt is a constant proportion of optimal capital, with the proportional factor given by

$$\ell^* = \max \left[0, \frac{1 - (f + \delta)(1 - \tau)}{1 + r_B (1 - \tau)} \right] . \tag{8}$$

This optimal book leverage ratio is the maximum book leverage consistent with risk-free debt, and it can be interpreted as the target leverage of the firm.[9] It is readily verified that ℓ^* is (strictly) less than 1, decreases in non-debt tax shields (i.e., operating costs f and depreciation δ) and the market cost of debt r_B, and is an increasing function of the income tax rate τ.

In expression (6), we observe that optimal capital increases with parameter α. As α goes up, the concavity of the production function with respect to capital input diminishes and the marginal productivity (and benefits) of capital increases.[10] The optimal capital also increases with the expected profit shock $E\left[z_{t+1}|z_t\right]$. In turn, the expected profit shock increases with the drift parameter c, current profit shock z_t, and volatility of innovations σ, which make the firm increase its optimal assets to take more advantage of their higher expected profits. The effect of the persistence parameter ρ on expected profit shocks depends on the value of current profit shock z_t. When $z_t > 1$, they grow with ρ, while the opposite is true when $z_t < 1$.[11] A negative effect on optimal capital level occurs with the five parameters in the denominator of W^*. Higher market costs of equity r_S, and debt r_B, income taxes τ, operating costs f, and depreciation δ increase the costs of capital and the firm diminishes optimal assets accordingly.

The next expression shows the market value of equity as an explicit solution of the model. It represents an analytic solution to the Gordon Growth Model in the stochastic setting, and it can thereby be used for valuation purposes.

Market Value of Equity

$$\widetilde{S}_t\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right) = \left[(1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta\widetilde{K}_t - r_B\widetilde{B}_t\right](1-\tau) + \widetilde{K}_t - \widetilde{B}_t + \widetilde{M}_t P^*. \qquad (9)$$

The first three terms on the right-hand side of equation (9) represent the (after-shock) book value of equity, while the last term is the going-concern value. Variable $\widetilde{M}_t$ is given by

$$\widetilde{M}_t = (1+g)^t e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} \left\{ \left(\frac{1+g}{1+r_S}\right) E\left[z_{t+1}^{1/(1-\alpha)}|z_t\right] + \left(\frac{1+g}{1+r_S}\right)^2 E\left[z_{t+2}^{1/(1-\alpha)}|z_t\right] + ... \right\} \qquad (10)$$

[9] The derivation of ℓ^* is in Lazzati and Menichini (2013), Appendix 1.

[10] The effect of α on optimal capital is positive for most values of the parameters, but it could be negative for some extreme combinations of paramaters.

[11] For standard values of the parameters, the invariant unconditional mean of profit shocks $E[z]$ is slightly above 1. Thus, the event $z < 1$ is fairly frequent.

and denotes the discounted sum of unconditional means of profit shocks. The general term of this infinite summation is

$$E\left[z_{t+n}^{1/(1-\alpha)}|z_t\right] = \left(c^{\frac{1-\rho^n}{1-\rho}} z_t^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{\left(1-\rho^{2n}\right)}{\left(1-\rho^2\right)} \frac{1}{(1-\alpha)}}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

and indicates the expected profit shock n periods from today given current innovation z_t.[12] Variable P^* is given by

$$P^* = \left(W^{*^{\alpha}} - fW^* - \delta W^* - r_B \ell^* W^*\right)(1-\tau) - r_S W^* (1-\ell^*) \tag{12}$$

and denotes the dollar return on equity minus the dollar cost of equity at the optimum.[13] Finally, factor $(1+g)^t$ in equation (10) represents the t periods of accumulated *past* growth while factor $(1+g)$ on the numerator of the discount factor in $\widetilde{M}_t$ introduces *future* growth into share price.

The next section shows that our model is able to rationalize several empirical findings in the cross-section of firms, and provides a structural explanation for each of them.

3 Structural Justification of the Main Empirical Findings

The empirical literature in financial economics has reported several regularities regarding firm decisions and outcomes. Some of these results appear to be inconsistent with the static trade-off theory of capital structure while some others have been used to question the assumption of perfect capital markets in the context of the neoclassical model of investment. The aim of this section is to show that our dynamic model captures fundamental aspects of firm behavior and, thus, can rationalize those critical findings in the cross-section of firms. The key step toward this

[12]We obtain $\widetilde{M}_t$ in the following way. Let $A_0 = 0$ and, for $n = 1, 2, \ ...,$

$$A_n = A_{n-1} + \left(\frac{1+g}{1+r_S}\right)^n E\left[z_{t+n}^{1/(1-\alpha)}|z_t\right].$$

Then, we iterate the previous recursion until convergence (i.e., until $A_n = A_{n-1} = A$). Finally, we compute $\widetilde{M}_t$ as

$$\widetilde{M}_t = (1+g)^t e^{-\frac{1}{2}\sigma^2 \frac{\alpha}{(1-\alpha)^2}} A.$$

[13]Lazzati and Menichini (2013) show that P^* is (strictly) positive.

achievement is to treat the relevant variables used in the literature as endogenous and to explain how their values depend on the different primitive characteristics of the firm.

In order to facilitate the reading of the following subsections, we describe those variables next.[14] We define book and market leverage in the usual way.

Book Leverage: $\ell_{b_t} = \frac{\widetilde{B}_t}{\widetilde{K}_t + \widetilde{N}_t}$ and **Market Leverage:** $\ell_{m_t} = \frac{\widetilde{B}_t}{\widetilde{B}_t + \widetilde{S}_t\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right)}$.

We introduce dividend payout as dividends over capital and investment ratio as investment over capital.

Dividend Payout: $d_{p_t} = \frac{\widetilde{L}_t}{\widetilde{K}_t + \widetilde{N}_t}$ and **Investment Ratio:** $i_t = \frac{\widetilde{K}_t - (1-\delta)\widetilde{K}_{t-1}}{\widetilde{K}_{t-1} + \widetilde{N}_{t-1}}$.

Regarding profitability, we follow the norm in the literature and define it as earnings before interest and taxes over capital.

Profitability: $p_t = \frac{\widetilde{E}_t}{\widetilde{K}_t + \widetilde{N}_t}$.

Then, we define internal cash flow as net profits plus capital depreciation over capital.

Internal Cash Flow: $h_t = \frac{\widetilde{N}_t + \delta \widetilde{K}_t}{\widetilde{K}_t + \widetilde{N}_t}$.

Finally, market-to-book ratio is defined as the market value of assets over their book value.

Market-to-Book Ratio: $q_t = \frac{\widetilde{B}_t + \widetilde{S}_t\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right)}{\widetilde{K}_t + \widetilde{N}_t}$.

Since all of the above variables are functions of the optimal policies, they are indeed endogenous in our analysis. The relationships between these variables and the primitive firm characteristics cannot be described by simple inspection, as we did in the previous section with the optimal policies. Given that these relationships are at the heart of our structural explanations,

[14] In order to make our conclusions comparable with the extant empirical work, we match the definition of these variables with the ones used by those studies. For instance, the leverage ratio ℓ^* in equation (8) represents the optimal level of leverage exactly after the firm makes the decision, but before the profit shock of the period is realized. However, it is unlikely that Compustat captures that ideal situation. To the extent that Compustat data are recorded at moments different from that of the decision, they will reflect the partial or total realization of the profit shock in the period. For this reason, we study the after-shock version of these variables.

we provide a sensitivity analysis in Appendix 2. One of the main results we obtain is that the curvature of the production function (α) and the non-debt tax shields ($f+\delta$) are the two features of the firm with the largest impact on the values of those endogenous variables.

In the following subsections we evaluate our model predictions regarding leverage, dividends, and investments.

3.1 Model Predictions About Leverage and Dividends

One of the main results reported by the empirical capital structure literature is the negative association between leverage and profitability. This inverse relationship usually appears in pooled ordinary least squares (OLS) regressions of leverage ratios and is one of the most persistent findings in the cross-section of firms. It has been used to challenge the trade-off theory of financing decisions with the following argument: under that theory, more profitable firms should have more leverage, after controlling for other effects, because their expected bankruptcy costs are lower. We next show that our dynamic model can rationalize this provocative finding. Moreover, by linking the endogenous variables in our model to the primitive characteristics of the firm, we provide a transparent justification for that negative association. We finally show our model can also explain other usual observed regularities regarding leverage and dividends.

In order to replicate the empirical findings, we first simulate the behavior of a heterogeneous group of firms (i.e., firms with different primitive characteristics) and then use pooled OLS regressions to study the associations between firm decisions and certain variables that are often treated as exogenous (e.g., profitability and market-to-book ratio).[15] We introduce firm heterogeneity into our analysis by using three SIC industries that display considerably different curvature of the production function and non-debt tax shields, which we show in Appendix 2 are among the most influential firm characteristics. In particular, we select Oil and Gas Extraction (OGE) as an industry with high capital elasticity and low non-debt tax shields, Chemicals (C) as an industry with an intermediate curvature of the production function and high non-debt tax shields, and Printing and Publishing (PP) as an industry with low capital elasticity and intermediate levels

[15] We use pooled OLS because it is one of the most common tools employed by the empirical capital structure literature to study the cross-section of firms.

of non-debt tax shields. We compute the model parameters for each industry using Compustat data and show their values in Table I. Lazzati and Menichini (2013) describe the procedure used to obtain those parameters for each industry.

[Insert Table I here]

Table II exhibits summary statistics for a representative firm from each of the three industries. We obtain these results after simulating each industry over 100,000 periods with the parameterization shown in Table I. We then use the last 100 observations from this simulation for each industry and do a pooled OLS regression of book leverage in order to study the model predictions regarding the cross-section of firms. For the regression, we use the standard specification

$$Y_{i,t} = \alpha + \beta X_{i,t-1} + \epsilon_{i,t} \tag{13}$$

where i indexes firms, t indexes time periods, Y denotes, alternatively, current values of book leverage, market leverage, and dividend payout, and X is a set of 1-period lagged values of profitability, market-to-book ratio, cash flow volatility, and dividend payout. (We compute cash flow volatility as the standard deviation of the last 10 periods of profitability.) Finally, $\epsilon_{i,t}$ is an *iid* random term.

[Insert Table II here]

The results from estimating equation (13) for book leverage, as well as for market leverage and dividend payout, are presented in Table III.

[Insert Table III here]

Regarding the regression of book leverage, the coefficient estimate on profitability is significantly negative, which implies that more profitable firms tend to have less leverage. In our model, this inverse association can be easily explained: First, Appendix 2 shows that the parameters that have the largest impact on book leverage, namely, the curvature of the production function (α) and the non-debt tax shields ($f + \delta$), are the same parameters with the greatest effect on profitability. Second, the impact of those parameters on book leverage is of opposite sign to that on profitability. Thus, in a cross-section of firms subject to different primitive features, the association between leverage and profitability will (most probably) be negative. Figure 1 shows this result graphically.

[Insert Figure 1 here]

Panel A displays book leverage versus profitability for each of the three industries, that is, OGE firms (square dots), C firms (dots with crosses), and PP firms (round dots). Of the three industries, OGE firms have the highest elasticity of capital and the lowest non-debt tax shields. This combination of parameters produces high leverage and low profitability on average. Conversely, C firms have lower elasticity of capital and higher non-debt tax shields. Accordingly, they have lower leverage and higher profitability on average. When these industries are pooled in an OLS regression, the negative association between these two endogenous variables naturally arises as a consequence of their dissimilar characteristics and the impact of these characteristics on the variables under study.

Table III also shows that book leverage is negatively associated with market-to-book ratio, cash flow volatility, and dividend payout. These results are also consistent with the empirical findings. As before, the elasticity of capital (α) and the non-debt tax shields ($f + \delta$) are among the dominant parameters for the last three variables. Furthermore, the effect of these parameters on those three variables is opposite to the effect on book leverage, which generates the negative relationship in a cross-section of firms. Panel B in Figure 1 shows graphically the negative relationship between book leverage and market-to-book ratio.

The results about market leverage are displayed on the third column in Table III, and are similar to those of book leverage.[16] The structural reasons behind these findings are similar as well, except that, for market leverage, other firm characteristics (i.e., the autoregressive coefficient (ρ) and the market discount rate of equity (r_S)) also play an important role. The last column of Table III shows the results for the dividend payout regression. The coefficient on profitability is positive, implying that more profitable firms tend to pay higher dividends. The structural

[16] The slightly negative slope on profitability is due to collinearity between profitability and dividend payout. Removing dividend payout from the regression makes the slope on profitability become strongly negative, as expected.

reason is that the curvature of the production function (α) and the non-debt tax shields ($f + \delta$) are the dominant parameters for these two endogenous variables and affect them in the same direction. Table III also shows that market-to-book ratio, cash flow volatility, and book leverage have significantly negative coefficients. Again, the underlying forces explaining these results are the opposite effects of the dominant parameters on the variables of interest.

We have shown the model we offer is able to rationalize many of the main regularities regarding leverage and dividend decisions. The next subsection explains a long-term puzzle regarding observed leverage decisions, namely, the fact that certain firms have no debt.

3.2 On the Existence of Zero-Leverage Firms

The existence of large, profitable, and stable firms that use zero debt is hard to explain in the context of the trade-off theory of financing decisions. According to that theory, they could issue some debt in order to shield earnings from income taxes and, thus, increase shareholder's wealth. In a recent paper, Strebulaev and Yang (2013) offer an in-depth study of zero-debt firms. They report that in the last decades, on average, these firms comprise around 10% of large public nonfinancial U.S. firms. They also find that those all-equity firms tend to be relatively more profitable, have higher market-to-book ratios, and pay higher taxes and dividends than control firms. Moreover, they suggest that this phenomenon is independent of firm size. **We next show that the dynamic model we offer can rationalize the existence of all-equity firms as well as their observed characteristics.**

A key aspect of our model is that it features a more conservative debt behavior as compared to the one implied by the trade-off theory. This feature is captured by the risk-free debt assumption we discussed earlier. Under risk-free debt, optimal leverage ℓ^* in equation (8) suggests that zero-leverage firms will be those with very high non-debt tax shields and low income tax rate. We can then rationalize zero-leverage decisions by simulating the behavior of a firm with those characteristics. In Appendix 2, we explain that, if we combine those characteristics with a low elasticity of capital (the main primitive features), the resulting firm will tend to have, in addition to zero-leverage, relatively high profitability, market-to-book ratio, dividend payout, and tax ratio (i.e., taxes over capital). Furthermore, because the drift in logs (c), which regulates the size of the firm, has been normalized to 1, these results hold irrespective of firm size. We can appreciate these predictions by comparing OGE versus C firms. Table I shows that the former have high elasticity of capital and low non-debt tax shields while the opposite is true for the latter. Accordingly, Table II shows that C firms have lower leverage (24.95% vs. 68.53%), higher profitability (39.29% vs. 15.10%), higher market-to-book ratio (8.40 vs. 3.20), higher dividend payout (27.49% vs. 6.95%), and higher tax ratio (8.93% vs. 3.47%) than OGE firms. These model predictions are consistent with the main findings of Strebulaev and Yang (2013).

Overall, our results suggest that the existence of zero-debt firms is not a puzzle and is consistent with shareholder value maximization under the restriction of risk-free debt. These findings possibly constitute the strongest evidence in favor of our dynamic model of the firm. Next, we describe the link between investment ratio, internal cash flow, and market-to-book ratio, and rationalize some results found by the empirical investment literature.

3.3 Model Predictions About Investments

The neoclassical intertemporal model of investment predicts that marginal q should be a sufficient statistics in investment regressions, that is, it should capture all relevant factors affecting investment decisions. However, a large body of empirical research shows that prediction almost never holds, since different measures of internal funds (e.g., output, sales, and internal cash flow) enter investment regressions as statistically significant regressors with a considerable explanatory power. Furthermore, the sensitivity of investment with respect to internal funds seems to be greater for those firms considered in principle to be more financially constrained, even after controlling for investment opportunities (e.g., Fazzari, Hubbard, and Petersen (1988)). These findings have been used to challenge the validity of the assumption of perfect capital markets. We next show that our dynamic model is able to rationalize these empirical observations. To this end, we estimate the corresponding regression with simulated data and show that the coefficient on internal cash flow can be easily significantly positive in investment regressions, even after controlling for marginal q. Most importantly, we show that firms that pay fewer dividends (i.e., the candidates to be considered more financially constrained) have larger investment-cash flow sensitivities, and vice versa.

Table II shows the mean dividend ratio (i.e., the proportion of net profits paid out as dividends) for the three SIC industries: OGE firms pay out a relatively low proportion of net income as dividends (58.52%) while C and PP firms pay out a relatively high fraction of net profits (91.43% and 98.11%, respectively).[17] Then, we do the following standard investment regression

$$i_{i,t} = \alpha + \beta_1 q_{i,t-1} + \beta_2 h_{i,t-1} + \epsilon_{i,t} \tag{14}$$

for each of the three types of firms and show the results in Table IV. First, we find that controlling for investment opportunities, the coefficient on internal cash flow is strongly positive in all investment regressions. This result is consistent with the long-standing empirical evidence. Second, the sensitivity of investment to internal cash flow is considerably higher for low dividend firms ($\beta_2 = 5.670$ for OGE firms) as compared to high dividend firms ($\beta_2 = 2.302$ for C firms and $\beta_2 = 3.492$ for PP firms). This finding coincides with the evidence presented by Fazzari, Hubbard, and Petersen (1988), though we obtain their result without any type of financial constraint. The structural force behind our finding is that the investment-cash flow sensitivity is strongly positively affected by the curvature of the production function (α), while the opposite effect holds for the dividend ratio. Thus, firms with high elasticity of capital (e.g., OGE firms) will pay low dividends and exhibit high investment-cash flow sensitivities, and vice versa. In addition, most of the other parameters have an opposite effect on those variables, reinforcing the inverse relation between them.

[Insert Table IV here]

Panel A in Figure 2 provides a clear graphical representation of these findings. We can see there that the points corresponding to the low dividend OGE firms (square dots) generate a much steeper slope than those of the high dividend C firms (dots with crosses) and PP firms (round dots).

[Insert Figure 2 here]

[17] The dividend ratio equals dividends over net profits, i.e., $d_{r_t} = \frac{\widetilde{L}_t}{\widetilde{N}_t}$. Net profits, $\widetilde{N}_t$, in the denominator is frequently close to zero in numerical simulations, which produces extreme values of the ratio and heavy-tailed numerical distributions. Therefore, we use the interquartile mean as a more robust measure of centrality.

4 Conclusion

Over time, the empirical side of the financial economics literature has documented a series of regularities regarding leverage, dividend, and investment decisions. Concomitantly, the theoretical side of the literature has been able to rationalize many of those results with different models (e.g., models of agency and asymmetric information) or assuming market imperfections (e.g., financing constraints). We provide a dynamic model of the firm that is able to rationalize most of those findings **simultaneously** without assuming frictions either on the real or the financing side. We show that the model we offer can explain, for instance, the observed negative association of leverage with profitability, the inverse relationship between dividend ratios and investment-cash flow sensitivities, as well as the existence of firms with zero debt and their observed characteristics. We believe the minimalist approach we employ, making explicit the endogeneity of the variables under study as well as their dependence on the primitive features of the firm, can also yield successful results in order to understand empirical findings in other areas of corporate finance.

References

[1] Ackerberg, Daniel A., C. Lanier Benkard, Steven Berry, and Ariel Pakes, 2007, Econometric tools for analyzing market outcomes, Handbook of Econometrics 6A, 4171–4276.

[2] Ackerberg, Daniel A., Kevin Caves, and Garth Frazer, 2006, Structural identification of production functions, Mimeo.

[3] Copeland, Thomas E., John F. Weston, and Kuldeep Shastri, 2005, Financial theory and corporate policy, Pearson Addison Wesley.

[4] DeAngelo, Harry, Linda DeAngelo, and Toni M. Whited, 2011, Capital structure dynamics and transitory debt, Journal of Financial Economics 99, 235–261.

[5] DeAngelo, Harry, and Ronald W. Masulis, 1980, Optimal capital structure under corporate and personal taxation, Journal of Financial Economics 8, 3–29.

[6] Easterbrook, Frank H., 1984, Two agency-cost explanations of dividends, American Economic Review 74, 650–659.

[7] Fama, Eugene F., and Kenneth R. French, 2002, Testing trade-off and pecking order predictions about dividends and debt, Review of Financial Studies 15, 1–33.

[8] Fama, Eugene F., and Merton H. Miller, 1972, The theory of finance, Holt, Rinehart and Winston, New York.

[9] Fazzari, Steven M., R. Glenn Hubbard, and Bruce C. Petersen, 1988, Financing constraints and corporate investment, Brookings Paper on Economic Activity 1, 141–195.

[10] Graham, John R., 2000, How big are the tax benefits of debt?, Journal of Finance 55, 1901–1941.

[11] Hennessy, Christopher A., and Toni M. Whited, 2005, Debt dynamics, Journal of Finance 60, 1129–1165.

[12] Hennessy, Christopher A., and Toni M. Whited, 2007, How costly is external financing? Evidence from a structural estimation, Journal of Finance 62, 1705–1745.

[13] Hubbard, R. Glenn, 1998, Capital-market imperfections and investment, Journal of Economic Literature 36, 193–225.

[14] Jaffee, Dwight M., and Thomas Russell, 1976, Imperfect information, uncertainty, and credit rationing, Quarterly Journal of Economics 90,651–666.

[15] Jensen, Michael C., 1986, Agency costs of free-cash-flow, corporate finance, and takeovers, American Economic Review 76, 323–329.

[16] Jensen, Michael C., and William H. Meckling, 1976, Theory of the firm: managerial behavior, agency costs and ownership structure, Journal of Financial Economics 3, 305–360.

[17] Kaplan, Steven N., and Luigi Zingales, 1997, Do investment–cash flow sensitivities provide useful measures of financing constraints?, Quarterly Journal of Economics 112, 169–215.

[18] Lazzati, Natalia, and Amilcar A. Menichini, 2013, A dynamic approach to the dividend discount model, Mimeo.

[19] Long, Michael S., and Ileen B. Malitz, 1985, Investment patterns and financial leverage, in B. J. Friedman (ed.), Corporate Capital Structures in the United States, University of Chicago Press, Chicago.

[20] Moyen, Nathalie, 2004, Investment-cash flow sensitivities: Constrained versus unconstrained firms, Journal of Finance 59, 2061–2092.

[21] Myers, Stewart C., 1977, Determinants of corporate borrowing, Journal of Financial Economics 5, 147–175.

[22] Myers, Stewart C., and Nicholas S. Majluf, 1984, Corporate financing and investment decisions when firms have information the investors do not have, Journal of Financial Economics 13, 187–221.

[23] Rajan, Raghuram G., and Luigi Zingales, 1995, What do we know about capital structure? Some evidence from international data, Journal of Finance 50, 1421–1460.

[24] Stiglitz, Joseph E., and Andrew Weiss, 1981, Credit rationing in markets with imperfect information, American Economic Review 71, 393–410.

[25] Strebulaev, Ilya A., and Baozhong Yang, 2013, The mystery of zero-leverage firms, Journal of Financial Economics 109, 1–23.

[26] Titman, Sheridan, and Roberto Wessels, 1988, The determinants of capital structure choice, Journal of Finance 43, 1–19.

[27] Tserlukevich, Yuri, 2008, Can real options explain financing behavior? Journal of Financial Economics 89, 232–252.

5 Appendix 1: Sketch of the Proof of the Solution of the Dynamic Model

In this appendix, we describe the main steps required to solve the firm problem in equation (5). We let vector $\widetilde{X}_t = \left\{\widetilde{K}_t, \widetilde{B}_t, \widetilde{E}_t, \widetilde{N}_t, \widetilde{L}_t, \widetilde{S}_t\right\}$ contain the model variables that grow over time. As a first step, we do the following normalization: $X_t = \widetilde{X}_t/(1+g)^t$. For instance, normalized capital is: $K_t = \widetilde{K}_t/(1+g)^t$. This normalization is required to keep the expectation of the payoff function in the future periods bounded. After the corresponding modification of the payoff function, we can reexpress the market value of equity as

$$S_0\left(K_0, B_0, z_0\right) = \max_{\{K_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{1+g}{1+r_S}\right)^t L_t \tag{15}$$

subject to the restriction of risk-free debt.

We solve this maximization problem recursively by using the Bellman equation associated to expression (15). We follow the norm in the literature and let normalized variables with primes denote values in the next period and normalized variables with no primes indicate current values (e.g., if we are at period t, then next-period assets are $K' = K_{t+1}$ and current assets are $K = K_t$). Then, we can write the Bellman equation for the problem of the firm in equation (15) as

$$S\left(K, B, z\right) = \max_{K', B'} \left\{L + \frac{(1+g)}{(1+r_S)} E\left[S\left(K', B', z'\right)|z\right]\right\} \tag{16}$$

subject to keeping debt risk-free. By using the standard backward induction arguments to solve equation (16), we find the following optimal policies for assets and debt, respectively,

$$K'^{*} = E\left[z'|z\right]^{\frac{1}{1-\alpha}} W^{*} \quad \text{and} \quad B'^{*} = \ell^{*} K'^{*} \tag{17}$$

where $E\left[z'|z\right] = c z^{\rho} e^{\frac{1}{2}\sigma^2}$, and W^* and ℓ^* are as described in equations (7) and (8), respectively.

6 Appendix 2: Sensitivity Analysis of Relevant Model Variables

In this appendix, we analyze how the asymptotic mean of the main model variables change when we vary the primitive characteristics of the firm. This sensitivity analysis allow us to understand the direction in which the fundamental parameters affect each of those variables as well as the magnitude of that effect. This analysis is important because the pooled OLS regressions in our study capture the long-run behavior of firms subject different primitive characteristics.

In order to perform this exercise, we start parameterizing the model described in the Section 2. Table V contains the values we use to parameterize the dynamic model for a representative firm. These values are common in the corporate finance literature. We normalize parameter c to 1. Following DeAngelo, DeAngelo, and Whited (2011), we set the autoregressive coefficient (ρ) at 0.75 and the standard deviation of the innovation term (σ) at 0.20. We fix the elasticity of capital input (α) at 0.65, which is close to the parameter estimate of Hennessy and Whited (2007). Finally, we set operating costs (f) equal to 0.20, the depreciation rate of capital (δ) equal to 0.10, the corporate income tax rate (τ) at 0.35, the market cost of debt (r_B) at 0.02 (which equals the risk-free interest rate r_f), the market cost of equity (r_S) at 0.08, and the secular growth rate (g) at 0.01. This parameterization refers to a period of a year.

[Insert Table V here]

Then, we simulate the model for 100,000 periods and study the long-run behavior of the main model variables (after discarding the first 100 observations). Table VI shows summary statistics of the stationary distributions of those variables. We finally study how the stationary mean of book leverage, $E(\ell_b)$, market leverage, $E(\ell_m)$, dividend payout, $E(d_p)$, profitability, $E(p)$, market-to-book ratio, $E(q)$, investment ratio, $E(i)$, and investment-cash flow sensitivity, β_2 in equation (14), vary when we change the base case parameter values by up to ± 20%. These numerical derivatives are useful to identify the parameters with the strongest effect on those variables (i.e., the dominant parameters).

[Insert Table VI here]

Table VII displays how the long-run mean book leverage changes with different parameter values. One of the most important parameters is the curvature of the production function (α). A 20% increment in the value of α (i.e., from 0.65 to 0.78) increases the asymptotic mean of book leverage from 70.21% to 73.86%. That is, the higher the elasticity of capital, the higher the expected book leverage in the long-run. Table VII also suggests that non-debt tax shields (e.g., the operating costs (f)) are important parameters. When f increases from 0.20 to 0.24, the stationary mean book leverage falls from 70.21% to 67.01%. The other parameters affect expected long-run book leverage to a lesser extent, with the corporate income tax rate (τ) and the capital depreciation rate (δ) at the top of this second group. In unreported results, we find that the sensitivity of the stationary mean of market leverage to the primitive parameters is quite similar to that of book leverage. In this case, besides the curvature of the production function and non-debt tax shields, the autoregressive parameter (ρ) and the market cost of equity (r_S) are also significant parameters.

[Insert Table VII here]

Table VIII exhibits the comparative statics analysis of the stationary mean of dividend payout. It is clear that the elasticity of capital input (α) has a significant influence on this variable. For the base case parameter value of $\alpha = 0.65$, $E\left(d_p\right)$ equals 10.00%, percentage that decreases to 2.06% when α goes up to 0.78. As before, non-debt tax shields (e.g., the operating costs (f)) also play an important role. When f goes up from the base case value of 0.20 to 0.24, mean dividend payout in the long-run increases from 10.00% to 11.13%. Table VIII also suggest that the volatility of profit shocks (σ) and the corporate income tax rate (τ) have a considerable impact on $E\left(d_p\right)$.

[Insert Table VIII here]

In Table IX, we study the effect of the different model parameters on the asymptotic mean of profitability $E\left(p\right)$. As is the case with book and market leverage, the two most significant parameters are the curvature of the production function (α) and non-debt tax shields ($f + \delta$). A 20% increase in α from 0.65 to 0.78 diminishes $E\left(p\right)$ from 19.32% to 12.33%, while when f goes up from 0.2 to 0.24, the long-run mean profitability increases from 19.32% to 21.12%. The other model parameters have a smaller effect on $E\left(p\right)$.

[Insert Table IX here]

Table X displays how different parameter values affect the stationary mean of market-to-book ratio, $E\left(q\right)$. In this case, the autoregressive coefficient (ρ), the elasticity of capital (α), and the market cost of equity (r_S) are the parameters with the largest impact on $E(q)$. Furthermore, the asymptotic mean of market-to-book ratio decreases in α while it increases in f. The other model parameters play a smaller role regarding this ratio.

[Insert Table X here]

Table XI exhibits the sensitivity analysis of mean investment ratio, $E(i)$, in the long-run. As with the other endogenous variables, the most important parameter is the curvature of the production function (α). An increase in α from 0.65 to 0.78 pushes $E(i)$ up from 16.80% to 33.45%. In this case, the standard deviation of the innovation term (σ) and the persistence of profit shocks (ρ) are also important parameters. Increasing σ 20% from 0.20 to 0.24 makes $E(i)$ go up from 16.80% to 20.15%, while augmenting ρ from 0.75 to 0.90 increases $E(i)$ from 16.80% to 19.46%. The other parameters have a smaller effect on expected long-run investment ratio.

[Insert Table XI here]

The sensitivity analysis of internal cash flow with respect to the different primitive parameters is quite similar to that of profitability in Table IX, so we omit it.

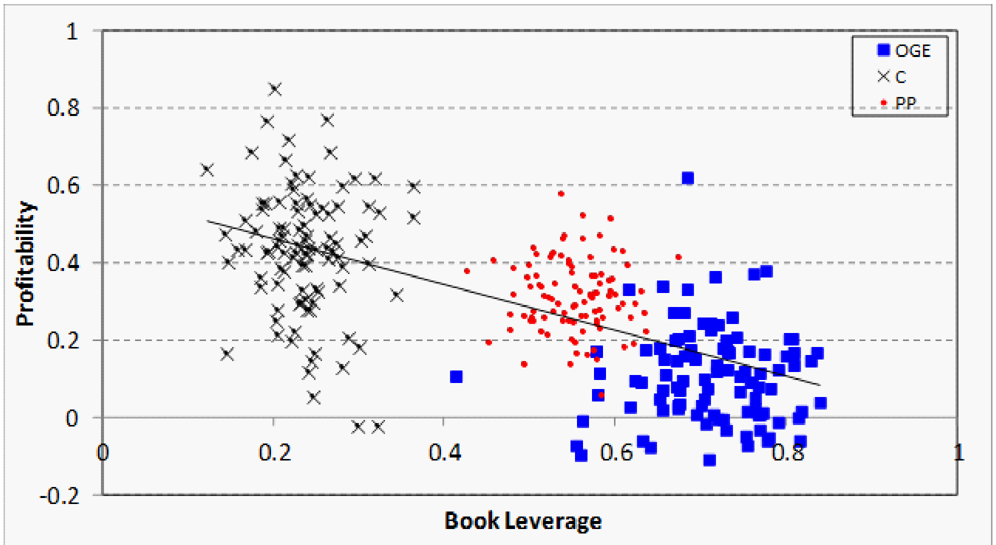
Finally, Table XII displays the sensitivity analysis for the investment-cash flow sensitivity (i.e., β_2 in equation (14)). By far, the most important parameter is the elasticity of capital (α). Increasing α 20% from 0.65 to 0.78 makes β_2 go up from 8.400 to 17.783. The other parameters affect β_2 to a lesser extent, with the non-debt tax shields and the autoregressive parameter at the top of this second group.

[Insert Table XII here]

The sensitivity analysis of dividend ratio is similar to that of dividend payout (so we omit it), with the curvature of the production function as the dominant parameter.

An important insight from this analysis is that variables frequently used as regressors in empirical work (e.g., profitability, market-to-book ratio, etc) are entirely endogenous and, jointly with the dependent variables, are affected by the same primitive parameters (mainly the curvature of the production function and non-debt tax shields). Then, acknowledging this fact and discovering their link to the structural parameters is a fundamental step in order to understand firm decisions in the cross-section.

Panel A: Book Leverage vs. Profitability



Panel B: Book Leverage vs. Market-to-Book Ratio

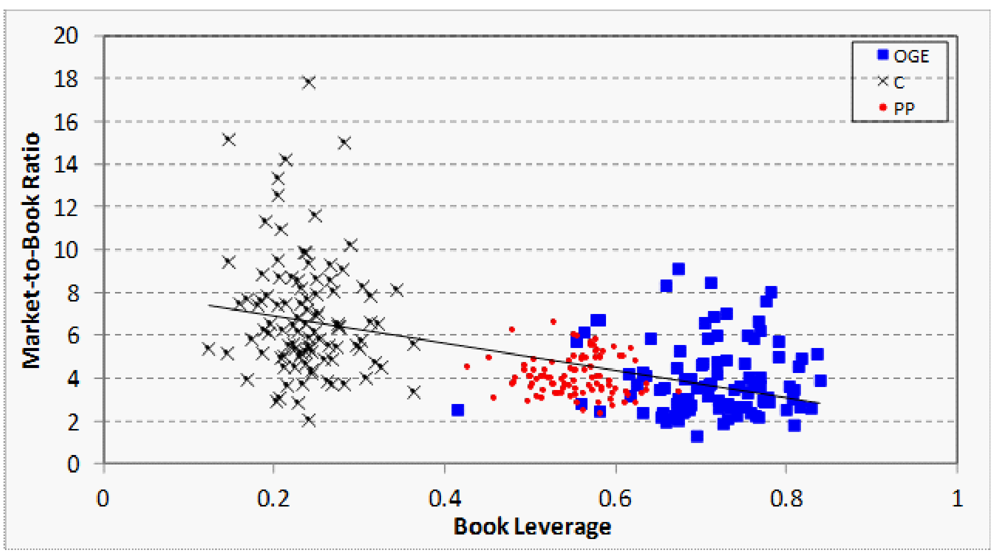
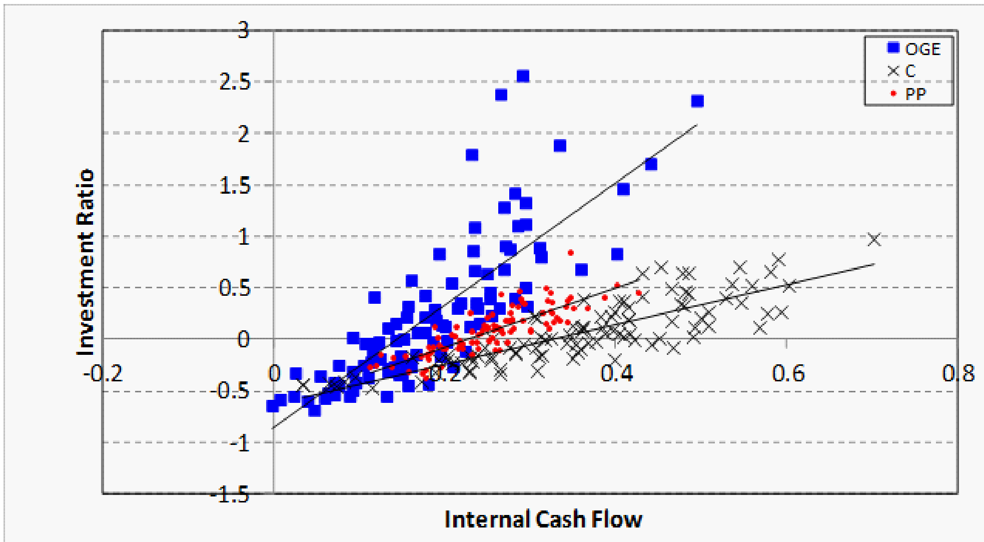


Figure 1. Book leverage versus profitability and market-to-book ratio. The model is simulated over 100,000 periods for each industry with the parameterizations described in Table I. The industries are Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). Panel A exhibits the last 100 observations of book leverage and profitability for each industry. Panel B shows the last 100 observations of book leverage and market-to-book ratio for each industry.

Panel A: Investment Ratio vs. Internal Cash Flow



Panel B: Investment Ratio vs. Market-to-Book Ratio

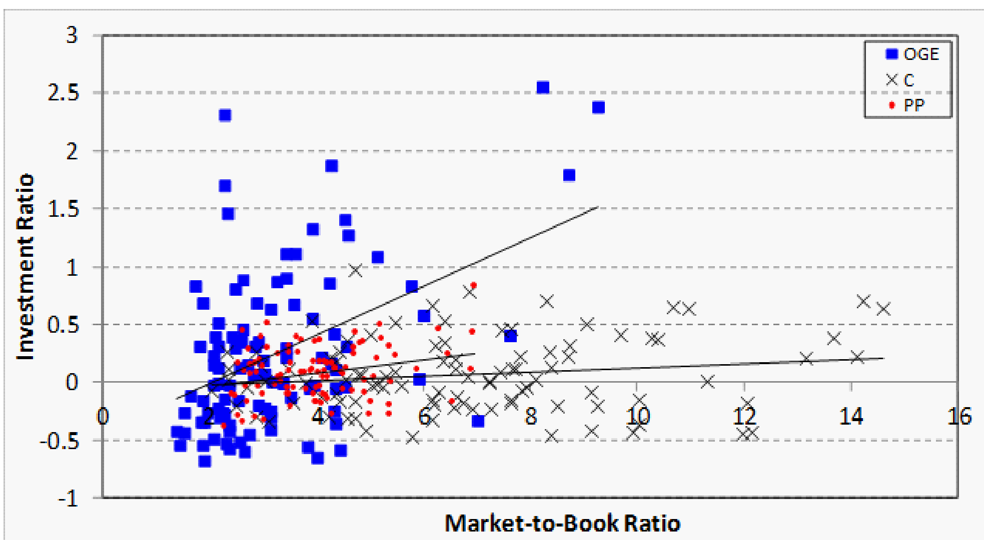


Figure 2. Investment ratio versus internal cash flow and market-to-book ratio. The model is simulated over 100,000 periods for each industry with the parameterizations described in Table I. The industries are Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). Panel A exhibits the last 100 observations of investment ratio and internal cash flow for each industry. Panel B shows the last 100 observations of investment ratio and market-to-book ratio for each industry.

Table I

Cross-Sectional Parameter Values

The table presents the values used to parameterize the dynamic model for three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

Parameter	*Value*		
	OGE	*C*	*PP*
c	1.0000	1.0000	1.0000
ρ	0.4748	0.5483	0.5603
σ	0.3633	0.2857	0.1787
α	0.6905	0.6146	0.5823
f	0.2079	0.7818	0.3746
δ	0.0925	0.0452	0.0617
τ	0.2519	0.2281	0.3210
r_B	0.0200	0.0200	0.0200
r_S	0.0710	0.0854	0.0902
g	0.0043	0.0347	0.0251

Table II

Cross-Sectional Summary Statistics

The table shows summary statistics from the simulation of the stationary distributions of relevant model variables for three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). The dynamic model is simulated over 100,000 periods for each industry with the parameterizations described in Table I. The variables are optimal leverage (ℓ^*), book leverage (ℓ_b), market leverge (ℓ_m), dividend payout (d_p), dividend ratio (d_r), investment ratio (i), profitability (p), internal cash flow (h), tax ratio (τ_r), market-to-book ratio (q), and profit shocks (z).

Variable	*OGE Firms*			*C Firms*			*PP Firms*		
	Mean	*Median*	*St. Dev.*	*Mean*	*Median*	*St. Dev.*	*Mean*	*Median*	*St. Dev.*
l^*	76.38%	76.38%	0.00%	35.61%	35.61%	0.00%	69.43%	69.43%	0.00%
l_b	68.53%	68.96%	7.62%	24.95%	24.52%	5.27%	55.18%	55.16%	4.37%
l_m	24.83%	24.16%	9.15%	3.50%	3.23%	1.48%	14.18%	13.90%	3.01%
d_p	6.95%	9.13%	9.85%	27.49%	29.29%	11.33%	19.48%	19.67%	3.26%
d_r	58.52%	59.22%	41.19%	91.43%	87.72%	25.95%	98.11%	96.17%	11.65%
i	22.26%	8.68%	67.01%	6.93%	5.35%	32.32%	8.29%	6.94%	21.65%
p	15.10%	14.37%	13.18%	39.29%	40.84%	19.07%	31.33%	31.37%	9.17%
h	18.57%	18.07%	9.05%	33.11%	34.26%	14.13%	25.42%	25.45%	5.90%
τ_r	3.47%	3.40%	3.35%	8.93%	9.30%	4.39%	9.84%	9.84%	2.98%
q	3.20	2.81	1.47	8.40	7.46	4.09	4.06	3.95	0.90
z	1.09	1.01	0.47	1.06	1.00	0.37	1.02	1.00	0.22

Table III

Leverage and Dividend Regressions

The table presents parameter estimates from pooled OLS regressions for a simulated sample of three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). The dynamic model is simulated over 100,000 periods for each industry with the parameterizations described in Table I. The regressions employ the last 100 observations from the simulation of each industry. The dependent variables are book leverage $\left(\ell_{b_{i,t}}\right)$, market leverge $\left(\ell_{m_{i,t}}\right)$, and dividend payout $\left(d_{p_{i,t}}\right)$, while the regressors are profitability $(p_{i,t-1})$, market-to-book ratio $(q_{i,t-1})$, cash flow volatility $\left(\sigma_{p_{i,t-1}}\right)$, book leverage $\left(\ell_{b_{i,t-1}}\right)$, and dividend payout $\left(d_{p_{i,t-1}}\right)$. The t-statistics appear in parentheses.

Variable	*Book Leverage*	*Market Leverage*	*Dividend Payout*
Intercept	0.944	0.299	0.504
	(67.76)	(29.74)	(16.99)
Profitability	-0.287	-0.004	0.205
	(-10.54)	(-0.18)	(7.69)
Market-to Book	-0.029	-0.018	-0.015
	(-18.60)	(-15.46)	(-13.26)
CF Volatility	-0.609	-0.027	-0.240
	(-6.08)	(-0.34)	(-4.79)
Book Leverage			-0.556
			(-17.86)
Dividend Payout	-0.790	-0.428	
	(-20.62)	(-15.44)	
R^2	0.840	0.674	0.867

Table IV

Investment Regressions

The table presents parameter estimates from OLS regressions for a simulated sample of three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). The dynamic model is simulated over 100,000 periods for each industry with the parameterizations described in Table I. The regressions employ the last 100 observations from the simulation of each industry. OGE firms pay a relatively low dividend ratio while C and PP firms pay a relatively high dividend ratio. The dependent variable is investment ratio $(i_{i,t})$ while the regressors are market-to-book ratio $(q_{i,t-1})$ and internal cash flow $(h_{i,t-1})$. The t-statistics appear in parentheses.

Variable	*OGE Firms*	*C Firms*	*PP Firms*
Intercept	-1.747	-1.169	-1.294
	(-34.35)	(-32.61)	(-68.28)
Market-to Book	0.248	0.059	0.119
	(23.94)	(20.18)	(42.62)
Internal Cash Flow	6.331	2.306	3.492
	(37.71)	(37.77)	(74.86)
R^2	0.950	0.938	0.984

Table V

Base Case Parameter Values

The table presents the values used to parameterize the base case of the dynamic model for a representative firm. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

Parameter	*Value*
c	1.00
ρ	0.75
σ	0.20
α	0.65
f	0.20
δ	0.10
τ	0.35
r_B	0.02
r_S	0.08
g	0.01

Table VI
Summary Statistics

The table shows summary statistics from the simulation of the stationary distributions of relevant model variables for a representative firm. The dynamic model is simulated over 100,000 periods (after discarding the first 100 observations) with the parameterization described in Table V. The variables are optimal leverage (ℓ^*), book leverage (ℓ_b), market leverge (ℓ_m), dividend payout (d_p), dividend ratio (d_r), investment ratio (i), profitability (p), internal cash flow (h), tax ratio (τ_r), market-to-book ratio (q), and profit shocks (z).

Variable	*Mean*	*Median*	*St. Dev.*
l^*	79.47%	79.47%	0.00%
l_b	70.21%	70.35%	4.13%
l_m	25.73%	25.23%	8.71%
d_p	10.00%	11.19%	4.89%
d_r	98.49%	96.46%	27.58%
i	16.80%	9.68%	44.03%
p	19.32%	19.06%	7.94%
h	20.48%	20.33%	4.70%
τ_r	6.27%	6.18%	2.81%
q	3.10	2.77	1.29
z	1.05	1.01	0.32

Table VII

Sensitivity Analysis of Book Leverage

The table shows the long-run mean of book leverage, $E(\ell_b)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$E(\ell_b)$	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$E(\ell_b)$	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(\ell_b)$	70.14%	70.15%	70.17%	70.19%	70.21%	70.21%	70.22%	70.25%	70.30%	70.34%	70.38%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$E(\ell_b)$	65.38%	66.48%	67.48%	68.44%	69.36%	70.21%	71.03%	71.78%	72.52%	73.20%	73.86%
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(\ell_b)$	73.52%	72.86%	72.19%	71.52%	70.87%	70.21%	69.55%	68.90%	68.26%	67.64%	67.01%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$E(\ell_b)$	71.86%	71.50%	71.20%	70.88%	70.52%	70.21%	69.87%	69.55%	69.26%	68.89%	68.61%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$E(\ell_b)$	67.46%	68.01%	68.57%	69.10%	69.68%	70.21%	70.76%	71.33%	71.88%	72.47%	73.02%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$E(\ell_b)$	70.48%	70.43%	70.37%	70.29%	70.25%	70.21%	70.15%	70.11%	70.08%	70.01%	69.95%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$E(\ell_b)$	70.52%	70.47%	70.39%	70.34%	70.24%	70.21%	70.16%	70.09%	70.04%	69.95%	69.89%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$E(\ell_b)$	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%	70.21%

Table VIII

Sensitivity Analysis of Dividend Payout

The table shows the long-run mean of dividend payout, $E(d_p)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$E(d_p)$	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$E(d_p)$	10.49%	10.39%	10.32%	10.20%	10.13%	10.00%	9.91%	9.80%	9.65%	9.59%	9.43%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(d_p)$	10.65%	10.53%	10.38%	10.28%	10.17%	10.00%	9.85%	9.70%	9.54%	9.34%	9.15%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$E(d_p)$	17.00%	15.51%	14.05%	12.67%	11.34%	10.00%	8.68%	7.26%	5.74%	4.06%	2.06%
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(d_p)$	8.85%	9.08%	9.32%	9.57%	9.77%	10.00%	10.24%	10.41%	10.64%	10.89%	11.13%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$E(d_p)$	9.42%	9.54%	9.64%	9.73%	9.87%	10.00%	10.11%	10.21%	10.35%	10.40%	10.54%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$E(d_p)$	10.97%	10.70%	10.59%	10.37%	10.18%	10.00%	9.82%	9.60%	9.43%	9.24%	9.04%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$E(d_p)$	9.91%	9.92%	9.93%	9.95%	9.97%	10.00%	10.02%	10.04%	10.06%	10.07%	10.08%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$E(d_p)$	9.62%	9.68%	9.74%	9.83%	9.89%	10.00%	10.09%	10.13%	10.24%	10.31%	10.36%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$E(d_p)$	10.05%	10.03%	10.02%	10.01%	10.00%	10.00%	10.00%	9.99%	9.98%	9.97%	9.96%

Table IX
Sensitivity Analysis of Profitability

The table shows the long-run mean of profitability, $E(p)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
E(p)	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
E(p)	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
E(p)	19.46%	19.44%	19.40%	19.36%	19.33%	19.32%	19.31%	19.26%	19.16%	19.08%	19.03%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
E(p)	28.58%	26.47%	24.56%	22.73%	20.95%	19.32%	17.75%	16.31%	14.90%	13.60%	12.33%
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
E(p)	17.43%	17.80%	18.19%	18.59%	18.95%	19.32%	19.71%	20.09%	20.44%	20.77%	21.12%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
E(p)	18.38%	18.61%	18.75%	18.92%	19.17%	19.32%	19.54%	19.71%	19.83%	20.11%	20.22%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
E(p)	19.01%	19.06%	19.10%	19.20%	19.22%	19.32%	19.41%	19.45%	19.54%	19.58%	19.69%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
E(p)	18.88%	18.95%	19.05%	19.19%	19.26%	19.32%	19.44%	19.50%	19.53%	19.65%	19.76%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
E(p)	18.73%	18.82%	18.99%	19.07%	19.26%	19.32%	19.42%	19.56%	19.66%	19.83%	19.93%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
E(p)	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%	19.32%

Table X

Sensitivity Analysis of Market-to-Book Ratio

The table shows the long-run mean of market-to-book ratio, $E(q)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$E(q)$	3.095	3.095	3.095	3.095	3.095	3.095	3.095	3.095	3.095	3.095	3.095
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$E(q)$	2.737	2.793	2.845	2.911	2.984	3.095	3.238	3.418	3.645	4.068	4.754
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(q)$	2.856	2.900	2.942	2.983	3.047	3.095	3.131	3.216	3.304	3.407	3.442
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$E(q)$	3.828	3.655	3.485	3.326	3.215	3.095	3.019	2.966	2.935	2.925	2.940
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(q)$	2.863	2.912	2.966	2.999	3.051	3.095	3.135	3.182	3.231	3.286	3.338
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$E(q)$	2.976	2.990	3.030	3.051	3.061	3.095	3.109	3.141	3.175	3.186	3.228
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$E(q)$	3.272	3.249	3.221	3.163	3.153	3.095	3.049	3.017	2.975	2.948	2.899
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$E(q)$	3.074	3.081	3.085	3.087	3.091	3.095	3.096	3.100	3.102	3.115	3.116
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$E(q)$	3.744	3.588	3.426	3.322	3.175	3.095	3.009	2.921	2.843	2.758	2.687
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$E(q)$	3.041	3.054	3.070	3.079	3.083	3.095	3.104	3.115	3.134	3.147	3.154

Table XI

Sensitivity Analysis of Investment Ratio

The table shows the long-run mean of investment ratio, $E(i)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$E(i)$	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%	16.80%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$E(i)$	14.39%	14.88%	15.33%	15.79%	16.23%	16.80%	17.26%	17.78%	18.36%	18.77%	19.46%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(i)$	14.17%	14.67%	15.16%	15.67%	16.23%	16.80%	17.39%	18.07%	18.64%	19.39%	20.15%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$E(i)$	11.82%	12.54%	13.33%	14.24%	15.43%	16.80%	18.52%	20.64%	23.70%	27.67%	33.45%
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$E(i)$	17.33%	17.20%	17.06%	16.95%	16.87%	16.80%	16.67%	16.60%	16.49%	16.44%	16.29%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$E(i)$	15.26%	15.64%	15.88%	16.26%	16.50%	16.80%	17.16%	17.43%	17.69%	18.09%	18.34%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$E(i)$	16.33%	16.45%	16.50%	16.67%	16.72%	16.80%	16.86%	16.99%	17.01%	17.13%	17.25%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$E(i)$	16.96%	16.93%	16.89%	16.86%	16.84%	16.80%	16.77%	16.75%	16.72%	16.70%	16.67%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$E(i)$	16.93%	16.90%	16.87%	16.84%	16.82%	16.80%	16.79%	16.77%	16.75%	16.73%	16.72%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$E(i)$	16.56%	16.63%	16.70%	16.73%	16.78%	16.80%	16.84%	16.87%	16.91%	16.95%	16.99%

Table XII

Sensitivity Analysis of Investment-Cash Flow Sensitivity

The table shows the regression coefficient on internal cash flow, β_2, from the investment regression in equation (14) for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table V while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
β_2	8.400	8.400	8.400	8.400	8.400	8.400	8.400	8.400	8.400	8.400	8.400
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
β_2	6.720	7.051	7.376	7.703	8.065	8.400	8.771	9.165	9.555	9.923	10.413
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
β_2	8.148	8.169	8.213	8.285	8.359	8.400	8.456	8.529	8.626	8.706	8.740
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
β_2	5.083	5.574	6.124	6.761	7.512	8.400	9.468	10.830	12.514	14.699	17.783
f	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
β_2	9.468	9.254	9.031	8.784	8.570	8.400	8.210	8.050	7.879	7.719	7.566
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
β_2	8.740	8.667	8.629	8.539	8.476	8.400	8.330	8.288	8.212	8.161	8.102
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
β_2	7.671	7.804	7.935	8.090	8.278	8.400	8.559	8.725	8.912	9.093	9.298
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
β_2	8.476	8.465	8.458	8.439	8.416	8.400	8.390	8.375	8.360	8.346	8.324
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
β_2	8.564	8.536	8.503	8.472	8.455	8.400	8.372	8.350	8.315	8.282	8.231
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
β_2	8.374	8.377	8.382	8.390	8.395	8.400	8.405	8.410	8.415	8.419	8.425